Time and frequency domain sampling principles

Time domain

For a vector of N samples in time domain, the RMS calculation is:

$$x_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} x_i^2}$$

Frequency domain

If Y is the FFT of x, where Y is a complex number. It contains N samples as well:

$$Y = FFT(x)$$

The RMS of Y (Parseval's theorem):

$$y_{RMS} = \frac{1}{\sqrt{N}} \sqrt{\frac{1}{N} \sum_{i=1}^{N} y_i^2}$$

Considering P1 as the normalized spectrum of N samples, and P2 the normalized reduced spectrum (of N/2 samples):

$$P 1_{i} = \frac{|Y_{i}|}{N}$$

$$P 2_{1} = P 1_{1}$$

$$P 2_{i} = 2P 1_{i} \quad \text{for } i \in (2:\frac{N}{2})$$

Where we have already taken modulus of each of Y's components. Then the corrected RMS of the normalized reduced spectrum equates to:

$$P 2'_{RMS} = \frac{\sqrt{N}}{2} \sqrt{\frac{1}{N/2} \sum_{i=1}^{N/2} P 2_i^2} = \frac{\sqrt{N}}{2} P 2_{RMS}$$

Conversion of RMS values to dB scale

Microphone values are referenced as peak values:

$$peak = RMS\sqrt{2}$$

The largest word size by the microphone is BITLENGHT = 24, so the maximum value it can read is $2^{BITLENGHT}$:

$$FULL SCALE (dB_{FS}) = 20 \log_{10} \left(2^{BITLENGTH} \right)$$

And the conversion of the peak values to FS (full scale) of the microphone:

$$P2_{RMS}(dB_{FS}) = 20\log_{10}(peak)$$

Hence, the SPL values, mapped to the sensor's range are:

$$P2_{RMS}(dB_{SPL}) = FULL SCALE(dB_{SPL}) - [FULL SCALE(dB_{FS}) - 20\log_{10}(peak)]$$

Where FULLSCALE(dBSPL) = 120 according to the manufacturer's data-sheet.

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